**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

The answer is (B) 0.2676

From the given data ,

*μ* = 45 *minutes*

*σ* = 8 minutes

X = 60-10 = 50 minutes

So calculating Z-score value for given data,

Z = (X - *μ*)/*σ = 0.625*

*From the Z table calculating the probability*

*Which is 0.7623*

*Hence the* probability that the service manager cannot meet his commitment *is given as*

*1 – 0.7323 = 0.2676*

In python same can be calculated as 1 - stats.norm.cdf(50,45,8),Hence the probability that the service manager cannot meet his commitment is given as = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

A.

It is false as from the Normal distribution plotted for the given data, the area of probability that people aging greater than 44 (area = 0.1587) is less the than the area of people aging between 38 and 44(area = 0.3413).

B.

Its true , because the probability of employees aging under 30 is 9.1 (i.e. area = 0.09176) and the percentage of 36 employees out of 400 employees aging under 30 is 9% which is equal to above probability.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Given that:

$1 = Rs. 45

Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

Thus,

**Company's profit**:

P ~ N( 5 + 7.32 + 42  ) = N(12,52)

A):

95% of the **probability** **lies**between 1.96 **standard deviations**of the **mean**.

Thus **range**is:

= 12 – 1.96 \*5 , 12 + 1.96 \* 5

= $2.2 M , $22.8 M

= Rs. 99M , Rs 1026 M

**Range**containing 95% **probability** for **profit**of **company**is

(Rs. 99M, Rs. 1026M).

B): **Fifth percentile**is calculated as:

P(Z <= (p-12)/5) = 0.05

From p **values**of z **score** **table**, we get:

(p – 12) / 5 = -1.644

P = 12- 8.22 = 3.78

Thus at $3.78M **dollars**, or Rs. 170.1M **amount**, 5th **percentile**of **profit** **lies**.

Or 5th **percentile**of **profit**is Rs. 170.1 Million.

C): Loss is when profit < 0

Thus: p < 0

The first **division** of **company**, thus have **larger probability**of making a loss in a given year.